Dominant spin-orbit effects in radiative decays $\Upsilon(3S) \to \gamma + \chi_{bJ}(1P)$

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We show that there are two reasons why the partial width for the transition $\Gamma_1(\Upsilon(3S) \to \gamma + \chi_{b1}(1P))$ is suppressed. First, the spin-averaged matrix element $\overline{I(3S|r|1P_J)}$ is small, being equal to 0.023 GeV^{-1} in our relativistic calculations. Secondly, the spin-orbit splittings produce relatively large contributions, giving $I(3S|r|1P_2) = 0.066$ GeV^{-1}, while due to a large cancellation the matrix element $I(3S|r|1P_1) = -0.020$ GeV^{-1} is small and negative; at the same time the magnitude of $I(3S|r|1P_0) = -0.063$ GeV^{-1} is relatively large. These matrix elements give rise to the following partial widths: $\Gamma_2(\Upsilon(3S) \to \gamma + \chi_{b2}(1P)) = 212$ eV, $\Gamma_0(\Upsilon(3S) \to \gamma + \chi_{b0}(1P)) = 54$ eV, which are in good agreement with the CLEO and BaBar data, and also to $\Gamma_1(\Upsilon(3S) \to \gamma + \chi_{b1}(1P)) = 13$ eV, which satisfies the BaBar limit, $\Gamma_1(\text{exp.}) < 22$ eV.

I. INTRODUCTION

In recent years, several new bottomonium states were discovered due to studies of radiative decays [1–4]. In [1] CLEO has observed the $\Upsilon(1D)$ in the four-photon decay cascade, $\Upsilon(3S) \to \gamma + \chi_b(2P)$, $\chi_b(2P) \to \gamma + \Upsilon(1D)$, $\Upsilon(1D) \to \gamma + \chi_b(1P)$, $\chi_b(1P) \to \gamma + \Upsilon(1S)$, and later this state was observed by BaBar in another four-photon cascade via the $\Upsilon(2S)$ [2]. In 2008 a new state, $\eta_b(1P)$, was discovered by BaBar, first in radiative decay $\Upsilon(3S) \to \gamma + \eta_b(1S)$ [3] and then in $\Upsilon(2S) \to \gamma + \eta_b(1S)$ [4]; later $\eta_b(1S)$ was confirmed

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by CLEO [5]. Moreover, new or more precise data on different radiative transitions, like $\Upsilon(3S) \to \gamma + \chi_b(n\,^3P_J)$ $(n = 1, 2)$, $\chi_b(1P, 2P) \to \gamma + \Upsilon(1S)$, and $\chi_b(2P) \to \gamma + \Upsilon(2S)$, were presented in Refs. [6–9].

This new experimental information is of a special importance for the theory to provide a better understanding of the role of relativistic and spin-dependent effects in bottomonium, and may be used as a test of different models and approximations. There are a large number of papers devoted to radiative decays in bottomonium [10–14], and a comparison of different results was already presented in [12–14], where the predicted partial widths are shown to be rather close to each other for most radiative E1 transitions and to agree with the existing experimental data. The only exception is the radiative decays $\Upsilon(3S) \to \gamma + \chi_b(1P_J)$ $(J = 0, 1, 2)$, which are discussed in detail in [14]. Their partial widths are defined by the matrix element (m.e.) $I(3S|r|1P_J) \equiv \langle \Upsilon(3S)|r|1\,^3P_J \rangle$ $(J = 0, 1, 2)$ and below we shall also use the spin-averaged m.e., denoted as $\overline{I(3S|r|1P)}$.

These m.e. strongly differ in the nonrelativistic (NR) and relativistic cases, even within the same model. The predicted transition rate $\Gamma_1(\Upsilon(3S) \to \gamma + \chi_{b1}(1P))$ varies in a wide range, $(3 - 110)$ eV [14] and is in many cases larger than the experimental width: $\Gamma_1 = (33 \pm 10)$ eV from the CLEO data [8]); a smaller value $\Gamma_1 = (10^{+8}_{-6})$ eV was measured by BaBar [9]. Moreover, even in the models which predict a small partial width Γ_1, their other two rates, $\Gamma_J(J = 0, 2)$, do not agree with the experimental values [15]. Therefore, the ratio of the transition rates, $r_{1,0} = \frac{\Gamma_1(\Upsilon(3S)\to\gamma+\chi_{b1}(1P))}{\Gamma_0(\Upsilon(3S)\to\gamma+\chi_{b0}(1P))}$, must be considered an important characteristic, which is small in experiments: $r_{1,0} \sim 0.5$ from the CLEO [8] and $r_{1,0} \sim 0.2$ from the BaBar data [9].

The m.e. $I(3S|r|1P_J)$ may differ several times in NR and relativistic calculations, even within the same model or while different static potentials are used [11, 16]. In Ref. [17] the suppression of this m.e. was shown to be quite strong in the NR limit for the power-law potentials $V(r) \sim r^\alpha$ with $-1 < \alpha < 2$. Since in bottomonium, even for $\Upsilon(3S)$, the relativistic corrections are not large, $\frac{\boldsymbol{p}^2}{m_b^2} \lesssim 0.1$, one may assume that this fact occurs because of the different asymptotics of the wave functions (w.f.) of the Schrödinger and relativistic equations.

An interesting result was obtained in Ref. [16], where for the NR Hamiltonian the partial width $\Gamma_2 = \Gamma(\Upsilon(3S) \to \gamma + \chi_{b2}(1P))$ decreases ten times, if instead of the Cornell potential with $\alpha(\text{static}) = $ constant, the Wisconsin potential which takes into account the asymptotic

freedom behavior of the vector strong coupling, is used. This result reminds of the situation with the dielectron widths of $\Upsilon(nS)$ ($n = 1, 2, 3$), where agreement with experiment is reached only for the potential with the asymptotic freedom behavior of the strong coupling [18].

However, even for this kind of potentials the spin-averaged m.e. $\overline{I(3S|r|1P)}$ appears to depend on the freezing (critical) value of the vector strong coupling used. In this paper we consider gluon-exchange (GE) potentials with two different values of α_{crit}.

It is also evident that since the m.e. $\overline{I(3S|r|1P_J)}$ is small, it may strongly depend on other small effects, in particular, on the spin-orbit interaction used. Here we show that due to the spin-orbit splittings the m.e. $I(3S|r|1P_J)$ acquire corrections of the same order as the value of the spin-averaged m.e. $\overline{I(3S|r|1P)}$, and a large cancellation takes place in the m.e. with $J = 1$. Here in our calculations we use the relativistic string Hamiltonian (RSH) [19], which was already tested in a number of papers, devoted to different bottomonium properties [20].

II.   RADIATIVE DECAYS

Electric dipole transitions between an initial state $i = 3\,^3S_1$, and a final state $f = 1\,^3P_J$, are defined by the partial width [10–14],

$$\Gamma(\ i \xrightarrow{\text{E1}} \gamma + f\ ) = \frac{4}{3}\, \alpha\, e_Q^2\, E_\gamma^3\, (2J' + 1)\, \mathrm{S}_{if}^{\mathrm{E}}\, |\mathcal{E}_{if}|^2 \ , \tag{1}$$

where $J' = J_f$, $l' = l_f$, and the statistical factor $\mathrm{S}_{if}^{\mathrm{E}} = \mathrm{S}_{fi}^{\mathrm{E}}$ is given by

$$\mathrm{S}_{if}^{\mathrm{E}} = \max(l, l') \left\{ \begin{matrix} J & 1 & J' \\ l' & s & l \end{matrix} \right\}^2 \tag{2}$$

and for the transitions between the $n\,^3S_1$ and $m\,^3P_J$ states with the same spin $S = 1$ this coefficient $S_{if}^E = 1/9$.

The RSH is simplified in the case of bottomonium, where in the Hamiltonian the string and self-energy corrections can be neglected because they are very small, ≤ 1 MeV. Then the original form of the RSH with the static potential

$$V_{\mathrm{B}}(r) = \sigma r - \frac{4}{3}\frac{\alpha_{\mathrm{B}}(r)}{r} \tag{3}$$

is

$$H = \frac{\boldsymbol{p}^2 + m_b^2}{\omega} + \omega + V_B(r). \tag{4}$$

Here m_b is the b-quark pole mass, while the value of ω is determined from the extremum condition $\frac{\partial H}{\partial \omega} = 0$, which gives $\omega = \sqrt{\boldsymbol{p}^2 + m_b^2}$, being equal to the kinetic energy of a b quark. Substituting this ω into Eq. (4) one arrives at the spinless Salpeter equation (SSE):

$$H_0 = 2\sqrt{\boldsymbol{p}^2 + m_b^2} + V_{\text{B}}(r), \tag{5}$$

The kinetic term occurring in (5) is widely used in relativistic potential models [21–23], however, as compared to constituent potential models, the RSH has several important differences.

1. By derivation, the mass of the b quark in the kinetic term cannot be chosen arbitrarily: it must be equal to the pole mass of a b quark, which takes into account perturbative in $\alpha_s(m_b)$ corrections. In two-loop approximations $m_b(\text{pole}) = \bar{m}_b(\bar{m}_b)(1 + 0.09 + 0.05)$ [24], where the second and third numbers come from the α_s and α_s^2 corrections, respectively. In our calculations $m_b(\text{pole}) = 4.83$ GeV is used, which corresponds to the conventional current mass $\bar{m}_b(\bar{m}_b) \simeq 4.24$ GeV.

2. H_0, as well as the mass $M(nl)$, does not contain an overall additive (fitting) constant.

3. The string tension $\sigma = 0.18$ GeV^2, used in the RSH, cannot be considered a fitting parameter, because it is fixed by the slope of the Regge trajectories for light mesons.

4. In the GE potential the asymptotic freedom behavior of the vector strong coupling $\alpha_{\text{B}}(r)$ is taken into account, being expressed via the "vector" QCD constant Λ_{B}, which is not a fitting parameter but defined by the conventional $\Lambda_{\overline{MS}}$ according to the relation: $\Lambda_{\text{B}}(n_f = 3) = 1.4753\,\Lambda_{\overline{MS}}(n_f = 3)$ and $\Lambda_{\text{B}}(n_f = 5) = 1.3656\,\Lambda_{\overline{MS}}(n_f = 5)$ [25]. On the other hand, the value of $\Lambda_{\overline{MS}}(n_f = 5)$ is fixed by the known value of $\alpha_s(M_Z)$ at the scale $M_Z = 91.19$ GeV. Here $\alpha_s(M_Z) = 0.1191$ is used, which in two-loop approximation gives $\Lambda_{\overline{MS}}(n_f = 5) = 240$ MeV and correspondingly, $\Lambda_{\text{B}}(n_f = 5) \simeq 330$ MeV.

Thus our scheme of calculations appears to be very restrictive in the case of bottomonium and only small variations of the fundamental parameters are admissible. However, some uncertainty comes from the value of the freezing constant, $\alpha_{\text{B}}(r \to \infty) \equiv \alpha_{\text{crit}}$, which

properties are discussed in Ref. [26]. Here we use the vector coupling in the range $0.49 \leq \alpha_{\mathrm{crit}} \leq 0.60$. Then for a given multiplet nl the centroid mass $M_{\mathrm{cog}}(nl)$ coincides with the eigenvalue $M(nl)$ of the SSE:

$$\left[2\sqrt{\boldsymbol{p}^2 + m_b^2} + V_{\mathrm{B}}(r)\right]\varphi_{nl} = M(nl)\varphi_{nl}. \tag{6}$$

For this relativistic equation the NR limit and the so-called *einbein* approximation may also be used and in both approximations a good description of the bottomonium spectrum is obtained, even for the higher states [20]. For most radiative decays (in bottomonium) the m.e. like $I(mS|r|nP_J)$ and $I(nP_J|r|mS)$ differ only by $10-20\%$ in the NR and relativistic cases, with the exception of the transitions $\Upsilon(3S) \to \gamma + \chi_{bJ}(1P)$. In this case our calculations give $\overline{I(3S|r|1P)} = 0.007 \text{ GeV}^{-1}$ in the NR case, being ~ 3 times smaller than $\overline{I(3S|r|1P)} = 0.023 \text{ GeV}^{-1}$ for the SSE (here $\alpha_{\mathrm{crit}} = 0.49$ was used). Notice that for a stronger GE potential with $\alpha_{\mathrm{crit}} = 0.60$ these spin-averaged m.e. appear to be larger: $\overline{I(3S|r|1P)} = 0.011 \text{ GeV}^{-1}$ in the NR case and 0.036 GeV^{-1} for the SSE.

Since the same static potential is used for the SSE as in the NR case, such a difference between the m.e. may be explained by two factors: the different asymptotic behavior of the w.f. of the SSE and Schrödinger equations, and also a smaller value of the w.f. at the origin for the Schrödinger equation as compared to that for the SSE. However, it is known that the w.f. $R_{nS}(r)$, as well as the derivative $R'_{1P}(r)$ for the $1P$ state, diverge near the origin for the SSE (these divergences are discussed in details in Ref. [16]) and the calculated values of the w.f. (or its derivative) at the origin are obtained with the use of a regularization procedure. This regularization introduces a theoretical error, which is estimated to be $\lesssim 10\%$.

In Table I we give the m.e. $I(3S|r|1P_J)$, calculated here for the SSE and in the NR limit, together with their values from second paper of Ref. [13] (table 4.16) for the NR and the relativistic variant RA, where a scalar confining potential, as in our calculations, is used.

Comparison of the m.e. presented in Table I shows that

1. In Ref. [13] for the relativistic variant RA the m.e. $I(3S|r|1P_J)$ is $\lesssim 4$ times larger than in the NR case; a similar result is obtained here for the spin-averaged m.e., where $\overline{I(3S|r|1P)} = 0.023 \text{ GeV}^{-1}$ for the SSE and is an ~ 3 times smaller value 0.007 GeV^{-1} in its NR limit.

2. Corrections $\delta I_{\mathrm{so}}(J) = I(3S|r|1P_J) - \overline{I(3S|r|1P)}$, due to the spin-orbit potential, have

TABLE I: The m.e. $I(3S|r|1P_J)$ (in GeV^{-1}) in the relativistic and NR cases

Transition	NR	RA$^{a)}$	NR$^{b)}$	SSE
	[13]	[13]	this paper	this paper
$\langle 3S|r|1P_2\rangle$	0.016	0.063	0.047	0.066
$\langle 3S|r|1P_1\rangle$	0.011	0.063	-0.033	-0.020
$\langle 3S|r|1P_0\rangle$	0.004	0.063	-0.073	-0.063

$^{a)}$Given numbers refer to the variant RA [13], where a scalar linear potential is used.

$^{b)}$ Given numbers refer to the NR limit of the SSE Eq. (6) with the same potential $V_{\mathrm{B}}(r)$ and $\alpha_{\mathrm{crit}} = 0.49$.

TABLE II: The partial widths $\Gamma(\Upsilon(3S) \to \gamma + \chi_b(1P_J))$ (in eV)

Transition	E_γ	RA	NR	SSE	exp.	exp.
	(MeV)	[13]	this paper	this paper	CLEO [6]	BaBar [8]
$\Gamma_2(\Upsilon(3S) \to \gamma + \chi_b(1P_2))$	433.5	195	108	213	157 ± 30	216 ± 25
$\Gamma_1(\Upsilon(3S) \to \gamma + \chi_b(1P_1))$	452.1	134	36	13	33 ± 10	< 22
$\Gamma_0(\Upsilon(3S) \to \gamma + \chi_b(1P_0))$	483.9	54	72	54	61 ± 23	55 ± 10

a relatively large value, e.g. $\delta I_{\mathrm{so}}(J = 2) = 0.043$ GeV^{-1}, being almost two times larger than $\overline{I(3S|r|1P)}$ in the spin-averaged case (see Eq. (9) below).

3. In Ref. [13] the splittings between the m.e. $I(3S|r|1P_J)$ with different J are much smaller than in our calculations.

4. In the spin-orbit potential we take the strong coupling $\alpha_{\mathrm{so}}(\mu) = 0.38$, which is close to the value $\alpha_{\mathrm{so}}(\mu(2P))$ used for the $\chi_{bJ}(2P)$ states (this value was extracted in Ref. [23] from the experimental masses of the members of the $\chi_{bJ}(2P)$ multiplet). Our calculations here show that the nondiagonal m.e., like $\langle nP|V_{\mathrm{so}}|mP\rangle$ $(n \neq m, n = 1, 2, 3)$, are of the same order or have even larger values than the diagonal m.e. $\langle 2P|V_{\mathrm{so}}(r)|2P\rangle$.

The calculated $E1$ transition rates are presented in Table II together with their values from Ref. [13]; they correspond to the m.e. from Table I.

In the relativistic case our transition rates appear to be very close to those from the BaBar data [9]. Even in the NR case, due to large spin-orbit corrections, the calculated

partial widths do not contradict the CLEO data [8].

We make some remarks on the contribution $\delta I_{\rm so}$ to the m.e. $I(3S|r|1P_J)$ from the spin-orbit potential, $\hat{V}_{\rm so}(r) = \boldsymbol{L} \cdot \boldsymbol{S}\ V_{\rm so}(r)$, for which the splittings $a_{\rm so}(nP|1P) = \langle nP|V_{\rm so}|1P \rangle,\ (n = 2, 3)$ are taken as for the one-gluon exchange interaction, i.e., neglecting the second order corrections in $\alpha_s(\mu)$ (it may be shown that the second order corrections are negative and small, ~ -0.7 MeV). In this approximation we find

$$a_{\rm so}(nP, 1P) = \frac{1}{2\omega_b^2}\{4\alpha_{\rm so}\langle r^{-3}\rangle_{nP,1P} - \sigma\langle r^{-1}\rangle_{nP,1P}\}, \tag{7}$$

where we take $\alpha_{\rm so} = 0.38$, which provided a good description of the fine-structure splittings for the $\chi_b(2P_J)$ multiplet. To determine the corrections to the w.f. of the $\chi_{bJ}(1P)$ states, the potential $\hat{V}_{\rm so}$ is considered as a perturbation and the following mass differences between the centroid masses are used:

$$M_{\rm cog}(2P) - M_{\rm cog}(1P) = 360 \text{ MeV},\ M_{\rm cog}(3P) - M_{\rm cog}(1P) = 640 \text{ MeV}. \tag{8}$$

Notice that the correction from the $3P$ state is not small, while the value of the centroid mass $M(3P)$, $M(\chi_b(3P)) \simeq 10.54$ GeV, is taken from the recent ATLAS experiment [27].

For the SSE, the splittings $a_{\rm so}(2P, 1P) = 12$ MeV and $a_{\rm so}(3P, 1P) = 10.2$ MeV, were calculated and in the NR limit their values are $\sim 10\%$ smaller. Then the nondiagonal m.e. $I(3S|r|1P_J)$ with the "spin-orbit" corrections can be presented (in GeV^{-1}) as

$$\begin{aligned} I(3S|r|1P_J) &= \overline{I(3S|r|1P)} + \delta I_{\rm so}(J), \\ \delta I_{\rm so}(J) &= 0.033\ \xi_J\ \overline{I(3S|r|2P)} + 0.016\ \xi_J\ \overline{I(3S|r|3P)}, \end{aligned} \tag{9}$$

where $\xi_J = -2, -1, +1$ for $J = 0, 1, 2$ and $\overline{I(3S|r|1P)} = 0.023$ GeV^{-1} for the SSE (relativistic case) and 0.007 GeV^{-1} in the NR limit. To obtain the m.e. presented in Table I, we use also the spin-averaged nondiagonal m.e.: $\overline{I(3S|r|2P)} = -2.54$ GeV^{-1} and $\overline{I(3S|r|3P)} = 2.64$ GeV^{-1}.

III. CONCLUSIONS

For the $E1$ radiative transitions, $\Upsilon(3S) \to \gamma + \chi_b(1P_J)\ (J = 0, 1, 2)$, the spin-averaged m.e. $\overline{I(3S|r|1P_J)}$ are shown to be small, as it was predicted in a number of studies before.

However, due to spin-orbit effects the w.f. of the $1\,{}^3P_1$ state is mixed with the $2P, 3P$ states, for which the m.e. $\overline{I(3S|r|2P)}$ and $\overline{I(3S|r|3P)}$ are large and have different signs.

Such a mixing is important, although the spin-orbit splittings themselves are not large and their typical values are $\sim 10 - 12$ MeV. Due to this mixing, a strong cancellation takes place in the m.e. $I(3S|r|1P_1)$, which gives rise to a suppression of the transition rate for the radiative decay $\Upsilon(3S) \to \gamma + \chi_b(1\,^3P_1)$.

The following partial widths are predicted: $\Gamma_J(\Upsilon(3S) \to \gamma + \chi_b(1\,^3P_J)) = 213$ eV, 13 eV, and 54 eV for $J = 2, 1, 0$, which are in good agreement with the BaBar data, $\Gamma_2(\text{exp.}) = 216 \pm 25$ eV and $\Gamma_0(\text{exp.}) = 55 \pm 10$ eV [9]. Also for $J = 1$ the calculated partial width $\Gamma_1 = 13$ eV satisfies the upper limit, $\Gamma_1 < 22$ eV, obtained in the BaBar experiment. More precise measurements of the transition rate for $\Upsilon(3S) \to \gamma + \chi_{b1}(1P)$ could give additional restrictions on the spin-orbit effects in radiative decays.

We predict the following ratio of the partial widths:, $r_{1,0} = \frac{\Gamma_1}{\Gamma_0} = 0.24$, which should be considered as an important feature of the transition rates where spin-orbit dynamics dominates.

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